Physics 4A
Chapter 7: Newton's Third Law

IMPORTANT CONCEPTS

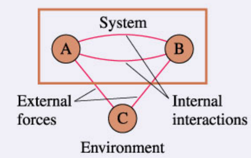
Objects, systems, and the environment

Objects whose motion is of interest are the system.

Objects whose motion is not of interest form the environment.

The objects of interest interact with the environment, but those interactions can be considered external forces.

Interaction diagram



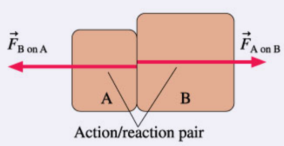
GENERAL PRINCIPLES

Newton's Third Law

Every force occurs as one member of an **action/reaction pair** of forces. The two members of an action/reaction pair:

- Act on two *different* objects.
- Are equal in magnitude but opposite in direction:

$$\vec{F}_{\text{A on B}} = -\vec{F}_{\text{B on A}}$$



Solving Interacting-Objects Problems

MODEL Identify which objects form the system.

VISUALIZE Draw a pictorial representation.

- Define symbols and coordinates.
- Identify acceleration constraints.
- Draw an interaction diagram.
- Draw a separate free-body diagram for each object.
- Connect action/reaction pairs with dashed lines.

SOLVE Write Newton's second law for each object.

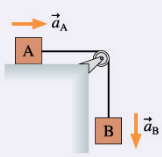
- Use the free-body diagrams.
- Equate the magnitudes of action/reaction pairs.
- Include acceleration constraints and friction.

ASSESS Is the result reasonable?

APPLICATIONS

Acceleration constraints

Objects that are constrained to move together must have accelerations of equal magnitude: $a_A = a_B$. This must be expressed in terms of components, such as $a_{Ax} = -a_{By}$.

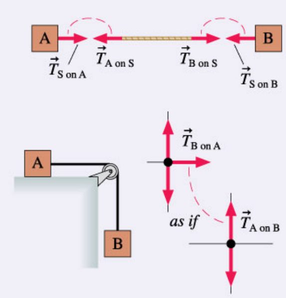


Strings and pulleys

The tension in a string or rope pulls in both directions. The tension is constant in a string if the string is:

- Massless, or
- In equilibrium

Objects connected by massless strings passing over massless, frictionless pulleys act *as if* they interact via an action/reaction pair of forces.

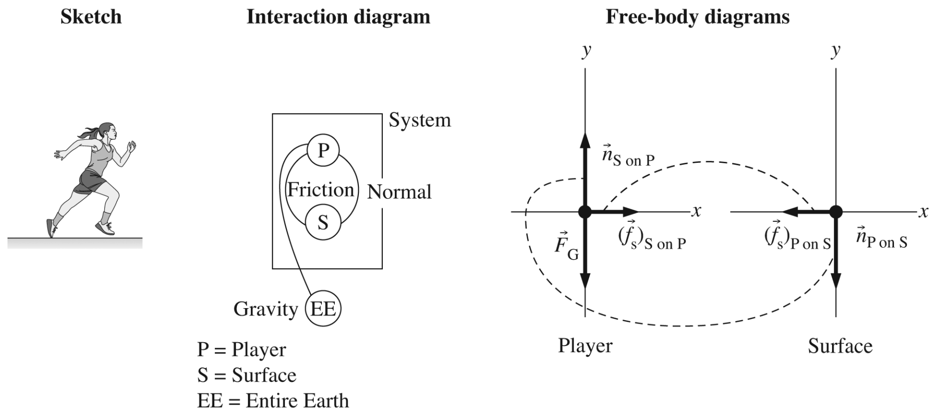


Conceptual Questions and Example Problems from Chapter 7

Conceptual Question 7.2

How does a sprinter sprint? What is the force on a sprinter as she accelerates? Where does that force come from?

7.2. The sprinter pushes backward on the ground, which pushes back (forward) on her. This is the only horizontal force on the sprinter, so she accelerates forward.



Conceptual Question 7.5

A mosquito collides head-on with a car traveling at 60 mph. Is the force of the mosquito on the car larger than, smaller than, or equal to the force of the car on the mosquito? Explain.

7.5. Newton's third law tells us that the force of the mosquito on the car has the same magnitude as the force of the car on the mosquito.

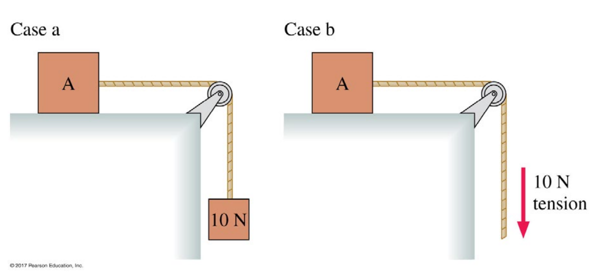
Conceptual Question 7.6

A mosquito collides head-on with a car traveling at 60 mph. Is the magnitude of the mosquito's acceleration larger than, smaller than, or equal to the magnitude of the car's acceleration?

7.6. The mosquito has a much smaller mass than the car, so the magnitude of the interaction force between the car and mosquito, although equal on each, causes the mosquito to have a much larger acceleration. In fact, the acceleration is usually fatal to the mosquito.

Conceptual Question 7.15

In case a in the figure below, block A is accelerated across a frictionless table by a hanging 10 N weight (1.02 kg). In case b, block A is accelerated across a frictionless table by a steady 10 N tension in the string. The string is massless, and the pulley is massless and frictionless. Is A's acceleration in case b greater than, less than, or equal to its acceleration in case a? *Explain.*

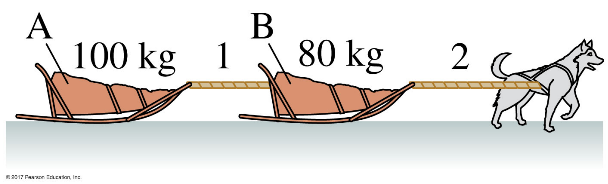


7.15. Block A's acceleration is greater in case b. In case a, the hanging 10 N must accelerate both the mass of A and its own mass, leading to a smaller acceleration than case b, where the entire 10 N force accelerates the mass of block A.

Case a	Case b
$10\text{ N} = (M_A + M_{10\text{ N}})a$	$10\text{ N} = M_A a$
$a = \dfrac{10\text{ N}}{(M_A + M_{10\text{ N}})}$	$a = \dfrac{10\text{ N}}{M_A}$

Problem 7.14

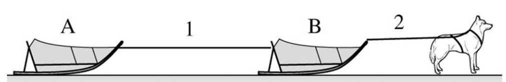
The sled dog in the figure below drags sleds A and B across the snow. The coefficient of friction between the sleds and the snow is 0.10. If the tension in rope 1 is 150 N, what is the tension in rope 2?



7.14. Model: Sled A, sled B, and the dog (D) are treated like particles in the model. The rope is considered to be massless.

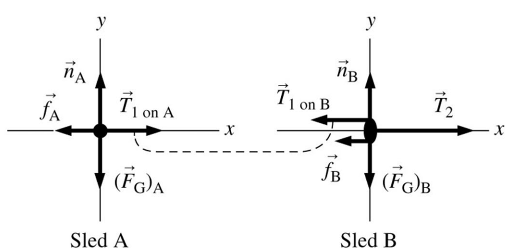
Visualize:

Pictorial representation



Known
$m_A = 100$ kg
$m_B = 80$ kg
$\mu_k = 0.10$
$T_1 = 150$ N

Find
T_2



Solve: The acceleration constraint is $(a_A)_x = (a_B)_x = a_x$. Newton's second law applied to sled A gives

$$\Sigma(\vec{F}_{\text{on A}})_y = n_A - (F_G)_A = 0\text{ N} \quad\Rightarrow\quad n_A = (F_G)_A = m_A g$$
$$\Sigma(\vec{F}_{\text{on A}})_x = T_{1\text{ on A}} - f_A = m_A a_x$$

Using $f_A = \mu_k n_A$, the x-equation yields

$$T_{1\text{ on A}} - \mu_k n_A = m_A a_x \quad\Rightarrow\quad 150\text{ N} - (0.10)(100\text{ kg})(9.8\text{ m/s}^2) = (100\text{ kg})a_x \quad\Rightarrow\quad a_x = 0.52\text{ m/s}^2$$

Newton's second law applied to sled B gives

$$\Sigma(\vec{F}_{\text{on B}})_y = n_B - (F_G)_B = 0\text{ N} \quad\Rightarrow\quad n_B = (F_G)_B = m_B g$$
$$\Sigma(\vec{F}_{\text{on B}})_x = T_2 - T_{1\text{ on B}} - f_B = m_B a_x$$

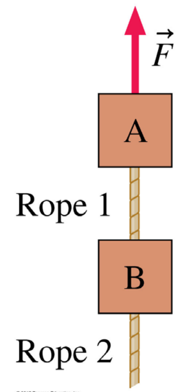
$T_{1\text{ on B}}$ and $T_{1\text{ on A}}$ act as if they are an action/reaction pair, so $T_{1\text{ on B}} = 150\text{ N}$. Using $f_B = \mu_k n_B = (0.10)(80\text{ kg})(9.8\text{ m/s}^2) = 78.4\text{ N}$, we find

$$T_2 - 150\text{ N} - 78.4\text{ N} = (80\text{ kg})(0.52\text{ m/s}^2) \quad \Rightarrow \quad T_2 = 270\text{ N}$$

Thus the tension $T_2 = 2.7 \times 10^2\text{ N}$.

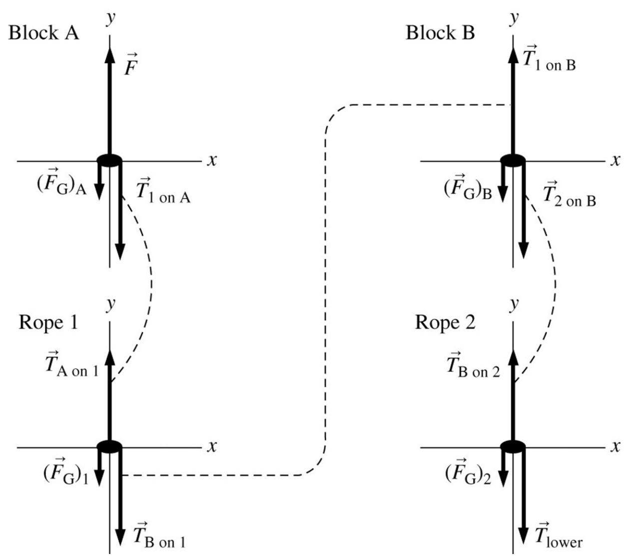
Problem 7.16

The figure to the right shows two 1.0 kg blocks connected by a rope. A second rope hangs beneath the lower block. Both ropes have a mass of 250 g. The entire assembly is accelerated upward at 3.0 m/s^2 by force F. **(a)** What is F? **(b)** What is the tension at the top end of rope 1? **(c)** What is the tension at the bottom end of rope 1? **(d)** What is the tension at the top end of rope 2?



7.16. Model: The two ropes and the two blocks (A and B) will be treated as particles.
Visualize:

Free-body diagrams



Solve: **(a)** The two blocks and two ropes form a combined system of total mass $M = 2.5$ kg. This combined system is accelerating upward at $a = 3.0\text{ m/s}^2$ under the influence of a force F and the gravitational force $-Mg\,\hat{j}$. Newton's second law applied to the combined system gives

$$(F_{\text{net}})_y = F - Mg = Ma \quad \Rightarrow \quad F = M(a+g) = (2.5\text{ kg})(3.0\text{ m/s}^2 + 9.8\text{ m/s}^2) = 32\text{ N}$$

(b) The ropes are *not* massless. We must consider both the blocks and the ropes as systems. The force F acts only on block A because it does not contact the other objects. We can proceed to apply the y-component of Newton's second law to each system, starting at the top. Each object accelerates upward at $a = 3.0\text{ m/s}^2$. For block A,

$$(F_{\text{net on A}})_y = F - m_A g - T_{1\text{ on A}} = m_A a \quad \Rightarrow \quad T_{1\text{ on A}} = F - m_A(a+g) = 19\text{ N}$$

(c) Applying Newton's second law to rope 1 gives

$$(F_{\text{net on }1})_y = T_{A\text{ on }1} - m_1 g - T_{B\text{ on }1} = m_1 a$$

where $\vec{T}_{A\text{ on }1}$ and $\vec{T}_{1\text{ on A}}$ are an action/reaction pair. But, because the rope has mass, the two tension forces $\vec{T}_{A\text{ on }1}$ and $\vec{T}_{B\text{ on }1}$ are *not* the same. The tension at the lower end of rope 1, where it connects to B, is

$$T_{B\text{ on }1} = T_{A\text{ on }1} - m_1(a+g) = 16\text{ N}$$

(d) We can continue to repeat this procedure, noting from Newton's third law that

$$T_{1\text{ on B}} = T_{B\text{ on }1} \text{ and } T_{2\text{ on B}} = T_{B\text{ on }2}$$

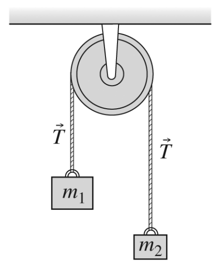
Newton's second law applied to block B is

$$(F_{\text{net on B}})_y = T_{1\text{ on B}} - m_B g - T_{2\text{ on B}} = m_B a \quad \Rightarrow \quad T_{2\text{ on B}} = T_{1\text{ on B}} - m_B(a+g) = 3.2\text{ N}$$

Problem 7.20

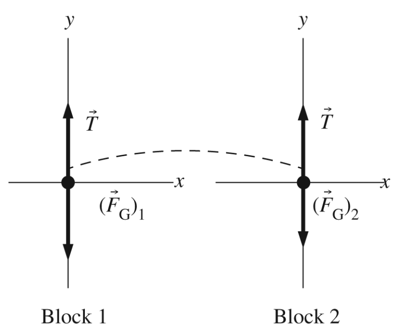
Two blocks are attached to opposite ends of a massless rope that goes over a massless, frictionless, stationary pulley. One of the blocks, with a mass of 6.0 kg, accelerates downward at ¾ g. What is the mass of the other block?

7.20. Model: The blocks are particles, the rope is massless, and the pulley is massless and frictionless.
Visualize: Because "the tension in a massless string remains constant as it passes over a massless, frictionless pulley" the tension will be the same everywhere in the rope. There is an acceleration constraint: $-a_1 = a_2 = a$.



Known
$m_1 = 6.0$ kg
$-a_1 = a_2 = a$

Find
m_2



Solve: First find the tension in the rope above the 6.0 kg mass. Then use that tension to find the mass of the other block.

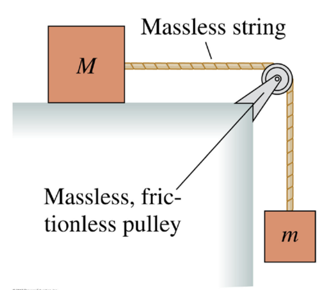
$$\Sigma F_1 = T - m_1 g = -m_1 a \Rightarrow T = m_1(g-a) = m_1\left(g - \tfrac{3}{4}g\right) = (6.0\text{ kg})\left(\tfrac{1}{4}g\right)$$

$$\Sigma F_2 = T - m_2 g = m_2 a \Rightarrow (6.0\text{ kg})\left(\tfrac{1}{4}g\right) - m_2 g = m_2\left(\tfrac{3}{4}g\right) \Rightarrow m_2 = \frac{(6.0\text{ kg})\left(\tfrac{1}{4}g\right)}{g + \tfrac{3}{4}g} = \tfrac{6}{7}\text{ kg} = 0.86\text{ kg}$$

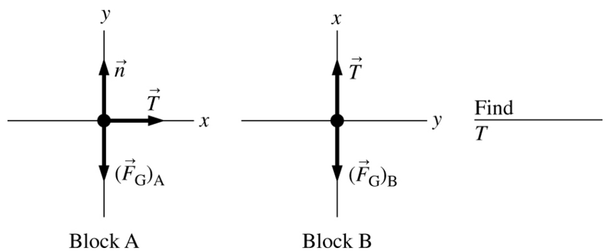
Since the tension is the same everywhere along the rope the woman must pull with a force of 250 N to keep a constant speed.
Assess: This can be checked by considering the two blocks as one compound object with a net force of $(m_1 - m_2)g$ and a total mass of $m_1 + m_2$.

Problem 7.36

The block of mass M in the figure to the right slides on a frictionless surface. Find an expression for the tension in the string.



7.36. Model: Blocks A and B make up the system of interest and will be treated as particles. There is no friction anywhere.
Visualize:



Notice that the coordinate system of for block B is rotated so that the motion in the positive x-direction is consistent between the two free-body diagrams.

Solve: The blocks are constrained to have the same magnitude acceleration. Applying Newton's second law to block B gives

$$\Sigma(F)_y = -T + (F_G)_B = ma \quad \Rightarrow \quad T - mg = -ma$$

Applying Newton's second law in both the x- and y-directions to the block A gives

$$\Sigma(F)_y = n - (F_G)_A = 0 \quad \Rightarrow \quad n = Mg$$
$$\Sigma(F)_x = T = Ma \quad \Rightarrow \quad T = Ma$$

Using the first equation to eliminate the acceleration a gives the tension:

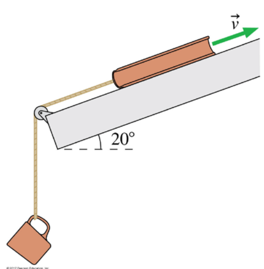
$$T = Ma = M(g - T/m) \quad \Rightarrow \quad T = \frac{mMg}{m+M}$$

Assess: The result is positive, as it should be for our choice of coordinate system. Consider $m = 0$. In this case, $T = 0$, as expected. For $m >> M$, the tension is independent of the mass of the hanging block because its acceleration will be g, as we can see by solving for the acceleration:

$$a = -\frac{T}{m} + g = g - \frac{Mg}{m+M} \rightarrow g \text{ for } m >> M$$

Problem 7.41

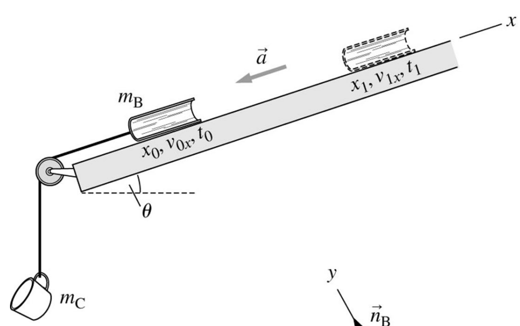
The 1.0 kg physics book in the figure below is connected by a string to a 500 g coffee cup. The book is given a push up the slope and released with a speed of 3.0 m/s. The coefficients of friction are $\mu_s = 0.50$ and $\mu_k = 0.20$. (a) How far does the book slide? (b) At the highest point, does the book stick to the slope, or does it slide back down?



7.41. Model: Use the particle model for the book (B) and the coffee cup (C), the models of kinetic and static friction, and the constant-acceleration kinematic equations.

Visualize:

Pictorial representation

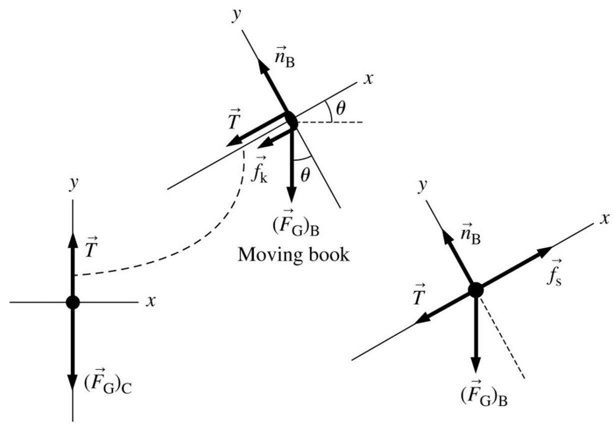


Known

$m_B = 1.0$ kg $\theta = 20°$
$m_C = 500$ g $v_{0x} = 3.0$ m/s
$t_0 = x_0 = 0$
$v_{1x} = 0$
$\mu_s = 0.50$ $\mu_k = 0.20$
$a_C = a_B = a$

Find

x_1



Coffee cup   Stationary book

Solve: (a) Using $v_{1x}^2 = v_{0x}^2 + 2a(x_1 - x_0)$, we find

$$0 \text{ m}^2/\text{s}^2 = (3.0 \text{ m/s})^2 + 2a(x_1) \quad \Rightarrow \quad ax_1 = -4.5 \text{ m}^2/\text{s}^2$$

To find x_1, we must first find a. Newton's second law applied to the book and the coffee cup gives

$$\Sigma(F_{\text{on B}})_y = n_B - (F_G)_B \cos(20°) = 0 \text{ N} \quad \Rightarrow \quad n_B = (1.0 \text{ kg})(9.8 \text{ m/s}^2)\cos(20°) = 9.21 \text{ N}$$

$$\Sigma(F_{\text{on B}})_x = -T - f_k - (F_G)_B \sin(20°) = m_B a_B \quad \Sigma(F_{\text{on C}})_y = T - (F_G)_C = m_C a_C$$

The last two equations can be rewritten, using $a_C = a_B = a$, as

$$-T - \mu_k n_B - m_B g \sin(20°) = m_B a \quad T - m_C g = m_C a$$

Adding the two equations gives

$$a(m_C + m_B) = -g[m_C + m_B \sin(20°)] - \mu_k(9.21 \text{ N})$$

$$(1.5 \text{ kg})a = -(9.8 \text{ m/s}^2)[0.500 \text{ kg} + (1.0 \text{ kg})\sin 20°] - (0.20)(9.21 \text{ N}) \quad \Rightarrow \quad a = -6.73 \text{ m/s}^2$$

Using this value for a, we can now find x_1 as follows:

$$x_1 = \frac{-4.5 \text{ m}^2/\text{s}^2}{a} = \frac{-4.5 \text{ m}^2/\text{s}^2}{-6.73 \text{ m/s}^2} = 0.67 \text{ m}$$

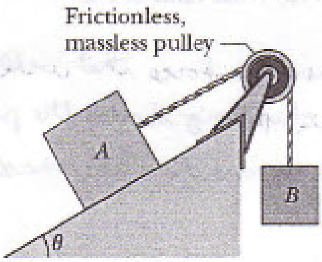
(b) The maximum static friction force is $(f_s)_{max} = \mu_s n_B = (0.50)(9.21 \text{ N}) = 4.60$ N. We'll see if the force f_s needed to keep the book in place is larger or smaller than $(f_s)_{max}$. When the cup is at rest, the string tension is $T = m_C g$. Newton's first law for the book is

$$\Sigma(F_{\text{on B}})_x = f_s - T - w_B \sin(20°) = f_s - m_C g - m_B g \sin(20°) = 0$$
$$f_s = (M_C + M_B \sin 20°)g = 8.25 \text{ N}$$

Because $f_s > (f_s)_{max}$, the book slides back down.

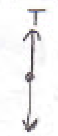
Problem 7.A (Problem from Exam 1 – Spring 2002)

In the figure below, two blocks are connected over a frictionless, massless pulley. The weight of block A is 50 N and the coefficient of static friction between block A and the incline is 0.20. The angle θ of the incline is 30.0°. What is the maximum and minimum mass that block B can have such that both blocks remain stationary?



⇒ the maximum mass block B can have means that block A is on the verge of slipping up the incline so $f_s = f_{s,max}$ and points down the incline

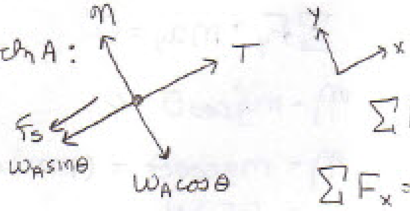
block B:



$\sum F_y = ma_y = 0 \quad T - w_B = 0$

$T = w_B \quad (1)$

block A:



$\sum F_y = ma_y = 0 \longrightarrow n = w_A \cos\theta$

$\sum F_x = ma_x = 0 \longrightarrow T - w_A \sin\theta - f_s = 0$

$T - w_A \sin\theta - f_{s,max} = 0 \longrightarrow T - w_A \sin\theta - \mu_s n = 0$

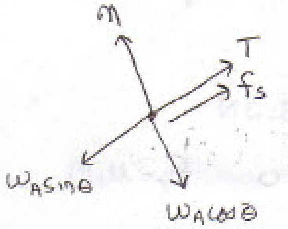
$T - w_A \sin\theta - \mu_s (w_A \cos\theta) = 0 \longrightarrow w_B - w_A \sin\theta - \mu_s (w_A \cos\theta) = 0$

$w_B = w_A \sin\theta + \mu_s w_A \cos\theta = (50N)\sin 30.0° + (0.20)(50N)\cos 30.0°$

$w_B = 33.7N \longrightarrow m = w/g = 33.7N / 9.80m/s^2 \longrightarrow \boxed{m_B = 3.4 Kg}$

⇒ the minimum mass block B can have means that block A is on the verge of slipping down the incline so $f_s = f_{s,max}$ and points up the incline

block A:



$\sum F_y = ma_y = 0 \longrightarrow n = w_A \cos\theta$

$\sum F_x = ma_x = 0 \longrightarrow T + f_s - w_A \sin\theta = 0$

$T = w_A \sin\theta - f_s \longrightarrow T = w_A \sin\theta - \mu_s n$

$w_B = w_A \sin\theta - \mu_s (w_A \cos\theta)$

$w_B = (50N)\sin 30.0° - (0.20)(50N)\cos 30.0° = 16.3N$

$m_B = w_B / g = 16.3N / 9.80m/s^2 \longrightarrow \boxed{m_B = 1.7Kg}$